MATH 170B: Discussion 4

Apr 2018

Fixed Point and Functional iteration

1. **Section 3.4.13** Let $p > 1$. Consider the following updating formula.

$$x = \cfrac{1}{p + \cfrac{1}{p + \cfrac{1}{p + \cfrac{1}{p + \dots}}}}$$

Note that this could be interpreted as $x_1 = \frac{1}{p}$, $x_2 = \frac{1}{p + \frac{1}{p}}$. $x_{n+1} = \frac{1}{p + x_n}$.

(1) What is the value of the following continued fraction?

(2) Developing an iterative method to solve for the roots of the quadratic equation $x^2 + px + q = 0$ $(p^2 - 4q \geq 0)$.

2. Show that the following functions are contractive on the indicated intervals. Determine the value of λ in equation $|f(x) - f(y)| < \lambda |x - y|$.

(1) $f(x) = 3 - x^2$ on $[-\frac{1}{4}, \frac{1}{4}]$.

(2) $g(x) = 2 - \sin(x)$ on the interval $[\frac{\pi}{6}, \frac{\pi}{2}]$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable such that $|f'(x)| < 1$ for all $x \in \mathbb{R}$. Show that f can have at most one fixed point.

Newton's method Computer Programming

4. **Computer problems Section 3.2.1** Write a computer program to solve the equation $x = \tan(x)$ by Newton's method. Find the roots nearest 4.5 and 7.7.