MATH 170B: Discussion 10

June 2018

Numerical integration

1. Suppose you have given the numerical integration scheme in the following

$$\int_0^1 f(x) = \frac{1}{90}\left[7f(0) + 32f(\frac{1}{4}) + 12f(\frac{1}{2}) + 32f(\frac{3}{4}) + 7f(1)\right]$$

Calculate $\ln 2$ approximately by applying the formla in the preceding problem to

$$\int_0^1 \frac{dt}{1+t}$$

2. Consider the given numerical integration scheme

$$\int_0^1 f(x) = w_1 f(0) + w_2 f(x_1)$$

Determine the values for w_1, w_2, x_1 that make the formula exact for all polynomials of degree as high as possible. What is the maximum degree?

3. Determine the constants a, b, c, d that will produce the given formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that is exact for polynomials of degree ≤ 3.