MATH 11: Discussion Week 9

May. 2019

Handout and its solution could be found at `https://kimukook.github.io/teaching/math11sp19/`

Confidence interval for mean 1. Suppose researchers capture and weigh 36 Anna's Hummingbirds. The weights of these 36 hummingbirds have an average of 4.6 grams with a standard deviation of 0.9 grams. Suppose that $t^*_{35} = 2.03$.
(a) Find a 95 percent confidence interval for the average weight of all Anna's Hummingbirds.
Solutions: The margin of error is

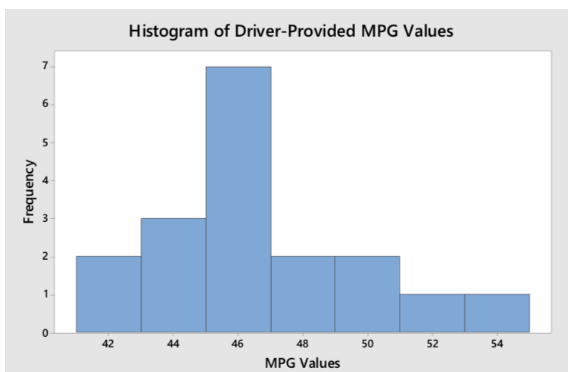
$$ME = t^*_{35}\frac{s}{\sqrt{n}} \approx 2.03\frac{0.9}{\sqrt{36}} = 0.3045$$

Therefore, a 95 percent confidence interval for the mean weight of all Anna's Hummingbirds is

$$(4.6 - .3045, 4.6 + .3045) \approx (4.3, 4.9).$$

(b) Do you think that approximately 95 percent of Anna's Hummingbirds have weights that fall in the interval that you computed in part a)? Briefly explain your answer.
Solutions: No, the confidence interval is for the mean weight of all Anna's Hummingbirds, not for the weights of individual hummingbirds.

Hypothesis testing for mean 2. The below data are a histogram of the MPG values for 18 people who drive a 2019 Toyota Prius. These data have a mean of 46.633 MPG with a standard deviation of 3.434 MPG. Toyota claims that the MPG of the 2019 Prius should be 48 MPG.



(a) You wish to conduct a test to determine if the average MPG of US drivers differs from Toyota's published value of 48 MPG. Write null and alternative hypotheses.
Solutions: Let μ be the mean MPG of all US 2019 Prius,

$$H_0 : \mu = 48$$
$$H_A : \mu \neq 48$$

(b) What conditions must be met in this setup in order to do inference? What concerns do you have about these conditions in this problem?
Solutions: We need:

- Independence. Here people self-select to report their data, so we are quite concerned about this condition.

- $< 10\%$ of population. Apparently this condition is satisfied.

- Sample is nearly normal.The histogram shows some right skew, but it is nearly normal.

(c) Assuming that the conditions in b) are met, what curve best approximates the sampling distribution?
Solutions: We will have a t-distribution with $df = 18 - 1 = 17$.

(d) Assuming that the conditions in b) are met, conduct your hypothesis test, give a P-value (or an interval containing the P-value, if that is the best you can do with your tables), and draw an appropriate conclusion using a significance level of 0.05.*(Hint: $P(T_{17} \leq -1.689) = 0.0547$)*
Solutions: Our test statistic is $t_n = \frac{46.633-48}{3.434/\sqrt{18}} \approx -1.689$. We have a two-sided test. Based on a t-table, the P-value is $2 * 0.0547 = 0.1095$, since this is bigger than 0.05, we do not reject H_0.

(e) From the data given in the problem, your friend calculates a 99% confidence interval. If you wanted an interval one-fourth the width of your friend, how big would the sample need to be?
Solutions: The width of ME is $t^*_{17} = t^*_{17}\frac{3.434}{\sqrt{n}}$. To cut this by $\frac{1}{4}$, we need $n = 16 * 18 = 288$.

Two sample t-test 3. Researchers performed an experiment to see whether the image of eyes watching would change employee behavior. They alternated pictures of eyes and looking at the viewer with pictures of flowers each week on the cupboard. They measured the consumption of milk and get the following result.

Category	Eyes	Flowers
n(weeks)	8	8
$\bar{y}$	0.417(liter)	0.151(liter)
s	0.1811	0.067

	90%	95%	97.5%	99%	99.5%	99.95%	1-Tail Confidence Level
	80%	90%	95%	98%	99%	99.9%	2-Tail Confidence Level
	0.100	0.050	0.025	0.010	0.005	0.0005	1-Tail Alpha
df	0.20	0.10	0.05	0.02	0.01	0.001	2-Tail Alpha
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9200	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3649	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	

(a) Find a 90% confidence interval for $\mu_1 - \mu_2$. What conclusion can you make?
Solutions: The margin of error is

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{0.1811^2}{8} + \frac{0.067^2}{8}} = 0.068$$

The degree of freedom is $df = \min(8-1, 8-1) = 7$, $t^*_7 = 1.8946$. The confidence interval is computed as

$$(\mu_1 - \mu_2) \pm ME = (0.1372, 0.3948)$$

We are 90% confident that the average difference between the comsumption of milk based on eyes images and floweres image is between 0.1372 and 0.3948.