MATH 11: Discussion Week 7

May 2019

Handout and its solution could be found at `https://kimukook.github.io/teaching/math11sp19/`

Mean and variance: 1. A company sells computer chips in packages of 100. They estimate that the average number of computer chips per package that will work is 90 with a standard deviation of 3 chips. **Five** separate packages are randomly selected for quality control.

(2 mins) (a) How many nonfunctional computer chips do you expect to find in total?
Solutions:
Let X_k denote the number of working ships in package k (for $k \in \{1, 2, 3, 4, 5\}$). Then by assumption, for each k,

$$\mathbb{E}[X_k] = 90, \ SD(X_k) = 3$$

Now let Y_k be the number of nonfunctional chips in package k (for $k \in \{1, ..., 5\}$). Then $Y_k = 100 - X_k$, for each k, and so:

$$\mathbb{E}[Y_k] = \mathbb{E}[100 - X_k] = 100 - 90 = 10$$
$$SD(Y_k) = SD(100 - X_k) = SD(X_k) = 3$$

Finally, let Y be the total number of nonfunctional chips. Then $Y = Y_1 + ... + Y_5$, and so:

$$\mathbb{E}[Y] = \mathbb{E}[Y_1 + ... + Y_5] = \mathbb{E}[Y_1] + ... + \mathbb{E}[Y_5] = 10 + ... + 10 = 50$$

(2 mins) (b) What is the standard deviation of the number of nonfunctional computer chips found? (Assume independence for the number of nonfunctional chips.)
Solutions:
Let Y and Y_k be the same concept defined in part (a). Then, by the independence assumption,

$$Var(Y) = Var(Y_1) + ... + Var(Y_5) = 3^2 + ... + 3^2 = 45$$

Therefore, $SD(Y) = \sqrt{Var(Y)} = 6.708$.

Sampling Distribution 2. A friend tells you that the average weight of American men is 191 pounds with a standard deviation of 23 pounds. Doubting this, you decide to draw a simple random sample of 300 American men and calculate the average. What is the probability you get an average less than 188 pounds? You do not need to check the conditions that we typically do for a problem like this.(Hint: $P(Z \leq -2.26) = 0.0119$)
Solutions:
The standard deviation is

$$SD = \frac{\sigma}{\sqrt{n}} = 23/\sqrt{300} = 1.3279$$

By CLT, the mean X of weights of 300 American men is $X \sim \mathcal{N}(191, 1.3279)$.
The z-score is $z = \frac{188-191}{1.3279} = -2.26$. Thus, the probability is 0.0119

Confidence Intervals:
Some concepts: **Standard error**: $SE(\hat{p})\sqrt{\frac{\hat{p}\hat{q}}{n}}$. **Margin of error**: $ME = z^* \cdot SE(\hat{p})$.
3. To approximate the percentage of people who self-identify as LGBT in Tokyo, we sample 800 volunteers from a local mall. Of these, 31 identify as LGBT.

(a) Find an 80% confidence interval for the true proportion of LGBT self-identifiers in Tokyo. (You do not need to check the conditions that we typically do for a problem like this.) (*Hint: For 80%*

confidence interval, $z^* = 1.28$.)
Solution:
We want to calculate $\hat{p} \pm z^* \cdot SE(\hat{p})$. It is obvious that $\hat{p} = \frac{31}{800} = .03875$ and $SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} \approx 0.0068235$. Thus we have

$$\hat{p} \pm z^* \cdot SE(\hat{p}) = (0.03875 + 1.28 \cdot 0.0068235, 0.03875 - 1.28 \cdot 0.0068235) = (0.03, 0.047).$$

(b) If we want to cut our margin of error in half (staying with 80% confidence), how large must the sample be? Provide some reasoning why your answer cuts the margin of error in half.
Solution:
If we take a sample four times as large, we have

$$SE = \sqrt{\frac{\hat{p}\hat{q}}{4n}} = \frac{1}{2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

If SE is cut in half, so is the margin of error. Thus we want $n = 4 * 800 = 3200$.

4. Suppose an investigator asked 510 randomly sampled adults the question about whether or not the death penalty is fair. Of these, 58% answered "Fairly", 42% said "Unfairly".

(3mins)(a). Find the 95% interval to contain the true proportion p, of adults who think the death penalty is applied fairly. (Remember to check for conditions.) Think about where does 1.96 come from, i.e. $P(z \leq 1.96) = 97.5\%$
Solution:
First check for 3 conditions, randamization condition and 10% condistion and success/failure condition. Then we can use a normal distribution to find a confidence interval. Second, since $n = 10$ and $\hat{p} = 0.58$, then $SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.58 \times 0.42}{510}} = 0.022$.
Because of the normal distribution model, the critical value $z^* = 1.96$.
Then $ME = z^* \times SE(\hat{p}) = 1.96 \times 0.022 = 0.043$.
Thus, the 95% confidence interval is 0.58 ± 0.043, or $(53.7\%, 62.3\%)$.

(3mins) (b) If we want to have a 2% margin of error(ME), how many adults should we choose in a new sample?
Solution:

Since $ME = z^* \times SE(\hat{p})$, we drive

$$0.02 = 1.96\sqrt{\frac{0.58 \times 0.42}{n}}$$

Then $n = \left(\frac{1.96\sqrt{0.58 \times 0.42}}{0.02}\right)^2 = 2339.53 \approx 2340$.

(3mins) (c) How to interpret this 95% interval?
Solution:
We're 95% confident that between 53.7% and 62.3% of all US adults think that the death penalty is applied fairly.

(3mins) (d) Find the 80% confidence interval to contain the true proportion p. Remind that $P(z \leq 1.28) = 90\%$
Solution:
First notice that $1 - \left(50\% - \frac{80\%}{2}\right) = 90\%$, the corresponding z-score of 90% is nearly 1.28 from the z-score table.
Thus $ME = z^* \times SE(\hat{p}) = 1.28 \times 0.022 = 0.028$. So the 80% confidence interval is 0.58 ± 0.028, or $(55.2\%, 60.8\%)$.