MATH 11: Discussion Week 6

Apr. 2019

Handout and its solution could be found at `https://kimukook.github.io/teaching/math11sp19/`

Geometric distribution 1. Suppose a student went to career fair booths, e.g. Google, Apple, Qualcomm, Texas Instruments, Motorola. And his/her likelihood of receiving an interview invitation depends on how well he/she did in Math 11. Suppose an A in Math 11 results in a probability $p = 0.95$ while C leads to $p = 0.15$.

(a) Give the probabiltiy density function for the random variable Y that denotes the number of career fair booth visits a student must make before his/her first invitation including the visit that results in the invitation. Express your answer in terms of p.
Solution:

$$p_Y(k) = p(1-p)^{k-1} \text{ for } k \geq 1$$

(b) On average, how many booth visits must an A student make before getting an interview invitation? How about a C student?
Solution:

$$\mathbb{E}[Y] = \frac{1}{p} = \begin{cases} 1.0526 & \text{A student} \\ 6.6667 & \text{C student} \end{cases}$$

(c) Assuming that each student visits 5 booths during a typical career fair, find the probability that an A student will not get an interview invitation.
Solution:
It can be observed directly that $P(\text{an A student does not get an invitation in 5 trials}) = (1-p)^5$ by the independence of each trial.

Possion distribution 2. Suppose the number of hits a web site receives in any time interval is a Poisson random variable. A particular site gets on average 5 hits per second.

(a) What is the probability that there will be no hits in an interval of two seconds?
Solution:
$\lambda = 2 * 5 = 10$

$$P(X = k) = e^{-10}\frac{10^k}{k!}$$

$$P(X = 0) = e^{-10}\frac{10^0}{0!} = e^{-10}$$

(b) What is the probability that there is at least one hit in an interval of one second?
Solution:
$\lambda = 5$, $X \sim \text{Poisson}(5)$,

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{e^{-5}5^0}{0!} = 1 - e^{-5}$$

Continuous random variable 3. Suppose $f(x) = \frac{1}{x^2}$, $\frac{1}{2} \leq x \leq 1$.
(1) What's the probability that $\frac{1}{2} \leq x \leq \frac{3}{4}$?
Solution:

$$\int_{\frac{1}{2}}^{\frac{3}{4}} \frac{1}{x^2}dx = -\frac{1}{x}\Big|_{\frac{1}{2}}^{\frac{3}{4}} = \frac{2}{3}$$

(2) What's the mean and variance of this random variable x on the interval $\frac{1}{2} \leq x \leq 1$?
Solution:
The mean:

$$\int_{1/2}^{1} xf(x)dx = \int_{1/2}^{1} \frac{1}{x}dx = \ln(x)\Big|_{1/2}^{1} = \ln 2 = 0.6931$$

The variance:

$$\begin{aligned}
&\int_{1/2}^{1} (x-\mu)^2 f(x)dx \\
&= \int_{1/2}^{1} (x^2 + (\ln 2)^2 - (\ln 4)x)\frac{1}{x^2}dx \\
&= \int_{1/2}^{1} 1dx + (\ln 2)^2 \int_{1/2}^{1} \frac{1}{x^2}dx - \ln 4 \int_{1/2}^{1} \frac{1}{x} \\
&= \frac{1}{2} + (\ln 2)^2 - \ln 4 \ln 2 = 0.0195
\end{aligned}$$

4. Using the z-score table provided, answer the following questions.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

Figure 1: z-score table

1. Suppose mean of midterm 1 is 22.21 with a standard deviation of 4.72. Now we try to use a normal distribution to approximate the scores.

(1) What's the probability that a student's score is above 27.0244?
Solution:
Let X denotes the score of student and Z denotes the corresponding z-score.

$$\begin{aligned}
P(X > 27.0244) &= 1 - P(X \leq 27.0244) \\
&= 1 - P(Z \leq \frac{27.0244 - \mu}{SD}) \\
&= \frac{27.0244 - 22.21}{4.72} \\
&= 1 - P(Z \leq 1.02) \\
&= 1 - 0.8461 = 0.1539
\end{aligned}$$

(2) What's the probability that a student's score is larger than 17.49 but smaller than 19.85?
Solution:

$$\begin{aligned}
P(X \leq 19.85) - P(X \leq 17.49) &= P(Z \leq \frac{19.85 - 22.21}{4.72}) - P(Z \leq \frac{17.49 - 22.21}{4.72}) \\
&= P(Z \leq -0.5) - P(Z \leq -1) \\
&= [1 - P(Z \leq 0.5)] - [1 - P(Z \leq 1)] \\
&= [1 - 0.6915] - [1 - 0.8413] \\
&= 0.1498
\end{aligned}$$