# MATH 11: Discussion Week 2

Apr. 2019

The handouts and solution could be found at `https://kimukook.github.io/teaching/math11sp19/`

## Properties of Mean and Variance

1. Suppose the temperatures $C$ measured in Celcius in March has mean and variance as $m(C) = 12$ and $\text{var}(C) = 4$. What is the corresponding mean and variance of $C$ transformed in Farenheit? *(Hint: $F = \frac{9}{5}C + 32$.)*
*Solution:* The mean would be

$$m(F) = \frac{9}{5}m(C) + 32 = 53.6$$

The variance would be

$$\text{Var}(F) = (\frac{9}{5})^2\text{Var}(C) = 12.96$$

2. __C_ In a meeting of a group of executives, the mean salary of the participants is \$100, 000, the median salary is \$80, 000, the upper quartile is \$125, 000, and the lower quartile is \$60, 000. Which of the following gives the best description of the distribution of salaries:
a) The distribution is most likely approximately symmetric.
b) The distribution is most likely skewed to the left.
c) The distribution is most likely skewed to the right.
d) The distribution is most likely bimodal.

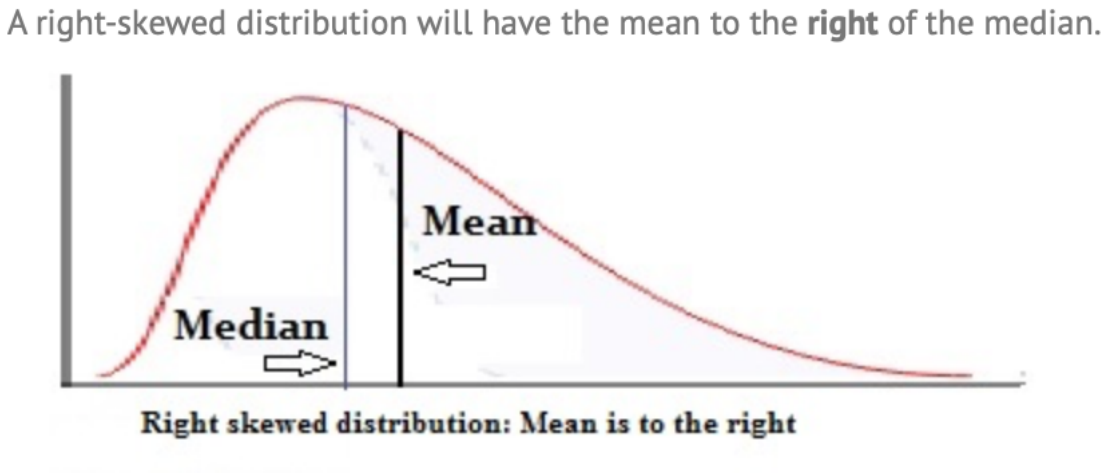



Figure 1: Explanation for Q2

# Correlation and linear regression

## 1. Correlation: A statistic that measures the Strength and Direction of a linear association (Form) between two quantitative variables where no Outliers are present.

(1) Determine which of the following statements about the correlation are **true** or **false**:

$$Hint: r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$$

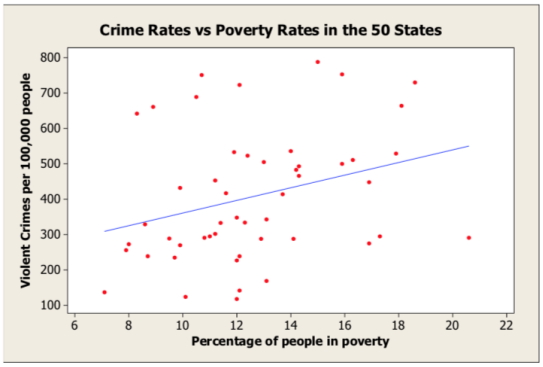
- **True** The range of correlation $r$ is $-1 \leq r \leq 1$.
- **True** The $+$ sign of correlation $r$ indicates the positive associations.
- **True** The stronger association occurs when $r$ is closer to 1 or $-1$.
- **True** The choice of predictor/response doesn't matter, i.e. $\text{cor}(X, Y) = \text{cor}(Y, X)$.
- **True** The correlation is unaffected by linear scale changes, i.e. $\text{cor}(X, Y) = \text{cor}(X, 2Y) = \text{cor}(X, Y + 5)$.
- **True** It is improper to measure correlation when even one single outlier is present.
- **True** It is improper to measure correlation for associations that are non-linear.

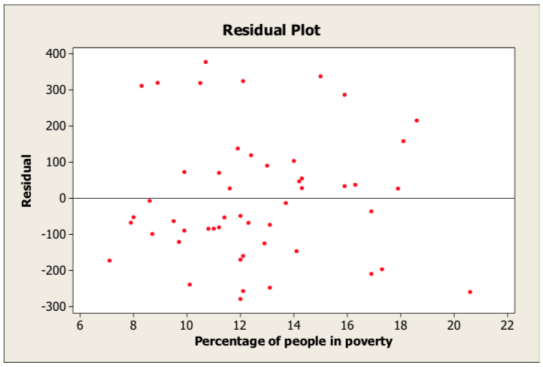
(2) Could you provide two examples to illustrate that correlation is not causation?
*Solution:*

- Weight and height are correlated (positive trend), but increasing weight doesn't mean that people can get taller.
- In the summer, the sales amount of ice-cream and the number of sunburns reported are both increasing, but doesn't mean that they have causal relation.

## 2. Linear regression

We have data on the poverty rate (percentage of people living in poverty) and crime rate (number of violent crimes per 100,000 people) in the 50 states. Below is some Minitab output for a regression in which the crime rate is the response variable and the poverty rate is the explanatory variable:
**Regression Analysis: Crime versus Poverty**
The regression equation is Crime = 182 + 17.9 Poverty
S = 176.889, R-Sq = 9.1%, R-sq(adj) = 7.2%.




(1) What would we predict to be the crime rate in a state where 16 percent of people live in poverty?
*Solution:* We predict the crime rate to be $182 + (17.9)(16) = 468.4$ violent crimes per 100,000 people.

(2) Is linear regression an appropriate way to predict the crime rate from the poverty rate? Explain your answer.

*Solution:* Yes, the residual plot shows a random scatter with no apparent patterns. There is no curvature, and there are no outliers. Therefore, regression is appropriate.

(3) Explain what the slope of the regression line means in this context.
*Solution:* A one percentage point increase in the number of people living in poverty is associated, on average, with an increase in the crime rate of 17.9 violent crimes per 100,000 people.

(4) If a state has a poverty that is one standard deviation above the mean, how many standard deviations above the mean would you predict the state's crime rate to be?
*Solution:* The correlation between the poverty rate and the crime rate is the square root of R-squared, which is $\sqrt{.091} \approx .302$. Therefore, if the state's poverty rate is one standard deviation above the mean, we predict the state's crime rate to be approximately .302 standard deviations above the mean.