MATH 11: Discussion Week 10

June, 2017

χ^2 for goodness-of-fit 1. In a certain genetics experiment, Mendel's theory predicts that 50 percent of snapdragon plants produced will be pink, 25 percent will be white, and 25 percent will be red. Suppose that, among 200 plants produced, we see that 94 are pink, 57 are white, and 49 are red. How well do these observations support the theory? Answer the question by conducting an appropriate hypothesis test. Carefully state your conclusion. *(Hint: The probability for χ^2 statistic bigger than 1.36 is 0.2533 with degree of freedom 2)*
Solution:

We conduct a chi-square test for goodness-of-fit. The null hypothesis is that plants are pink, white, and red with probabilities 0.5, 0.25, and 0.25 respectively. The alternative hypothesis is that the colors arise in some other proportions. If the null hypothesis is true then we expect (200)(0.5) = 100 pink flowers, and (200)(0.25) = 50 white and red flowers. Since we observed 94 pink flowers, 57 white flowers, and 49 red flowers, our chi-square test statistic is

$$\chi^2 = \frac{(94-100)^2}{100} + \frac{(57-50)^2}{50} + \frac{(49-50)^2}{50} = 1.36$$

If H_0 is true, this test statistic has a chi-square distribution with 3 - 1 = 2 degrees of freedom. From the table, we see that the p-value is greater than 0.10, so we fail to reject H0. The data give us no reason to doubt Mendel's theory.

Regression analysis 2. The body temperatures (in degrees Fahrenheit) and heart rates (in beats per minute) of 130 people were recorded. We are interested in investigating the relationship between heart rate and body temperature, and in predicting the heart rate of someone whose body temperature is 98.0. Below is regression output from MINITAB along with some relevant graphs.
Regression Analysis: HeartRate versus BodyTemp
The regression equation is
HeartRate $= -166 + 2.44$BodyTemp

Predictor	Coef	SE Coef	T	P
Constant	-166.28	80.91	-2.06	0.042
BodyTemp.	2.4432	0.8235	2.97	0.004

S = 6.85774 R-sq = 6.4%

Predicted values for New Observations

New Obs	Fit	SE Fit	95% CI	95% PI
1.	73.153	0.636	(71.895, 74.410)	(59.525, 86.780)

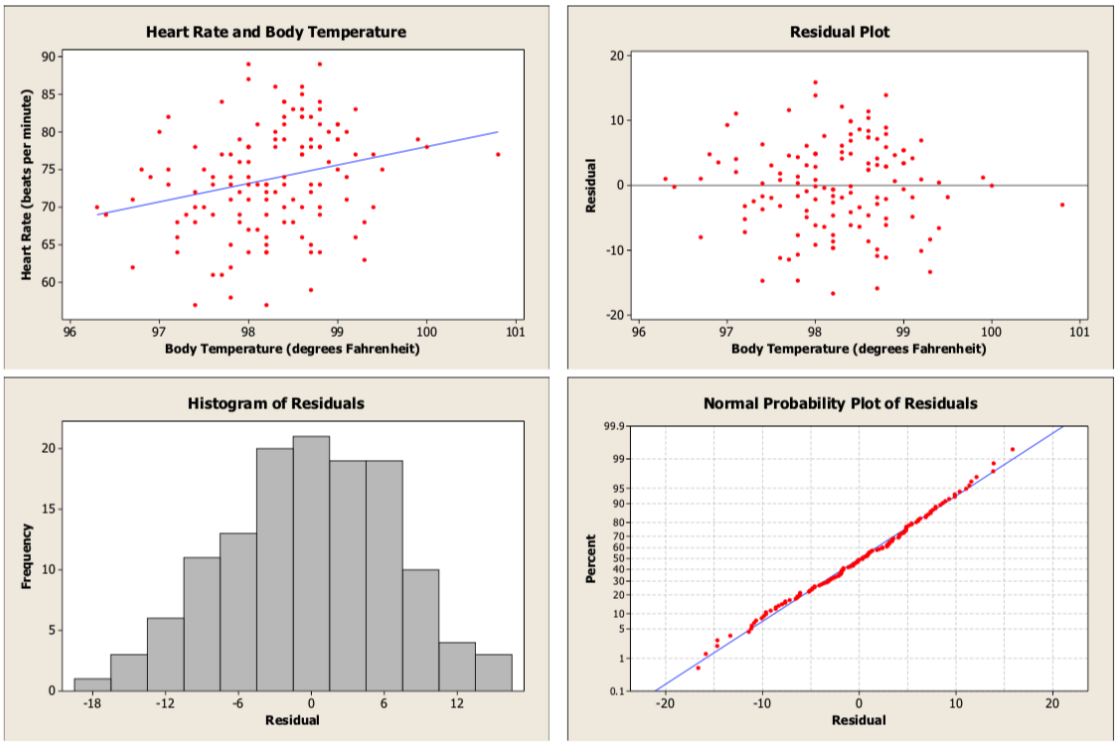

Figure 1: Caption

a) Are the assumptions for regression inference satisfied? Explain your answer.
Solution:
Yes, the residual plot shows a random scatter with no curvature, outliers, heteroskedasticity, or other patterns. Also, the residuals are approximately normally distributed.

b) What percentage of the variation in heart rate can be explained by differences in body temperature?
Solution:
This is the R-Squared, which is 6.4 percent.

c) Is there strong evidence of an association between heart rate and body temperature? Explain your answer. Use significance level 0.05.
Solution:
Yes, the p-value for the test of $H_0 : \beta_1 = 0$ against $H_A : \beta_1 \neq 0$ is 0.004, which is less than 0.05. Therefore, we reject the null hypothesis, which means that we have strong evidence that the slope of the true regression line is positive and thus that there is an association between heart rate and body temperature.

d) Find a 95 percent confidence interval for the slope of the true regression line. *(Hint:* $t^*_{128} = 1.98$ *for 95% CI)*
Solution:
The margin of error is

$$ME = t^*_{128} SE(b_1) \approx (1.980)(0.8236) \approx 1.63$$

so a 95 percent confidence interval for the slope of the true regression line is

$$(2.4432 - 1.63, 2.4432 + 1.63) \approx (0.87, 4.61)$$

e) Find an interval which you are 95 percent confident will contain the heart rate of a person whose body temperature is 98.0.
Solution:
This is a 95 percent prediction interval, which is reported as (59.525, 86.780).